Score: ________ / 40

UNIT #11 ASSESSMENT
COMMON CORE ALGEBRA II

Part I Questions

1. When drawn in standard position, which of the following angles is coterminal with 215°?

(1) -215° (3) 595°

(2) -505° (4) 915°

Angle that are coterminal will differ by integer multiples of $360°$:

$$215° - 360° - 360° = -505°$$

(2)

2. Which of the following angles, in radians, is equivalent to 285°?

(1) $\frac{19\pi}{12}$ (3) $\frac{65\pi}{57}$

(2) $\frac{17\pi}{15}$ (4) $\frac{16\pi}{15}$

$$285° \times \frac{\pi}{180°} = \frac{(285 \div 15)\pi}{180 \div 15} = \frac{19\pi}{12}$$

(1)

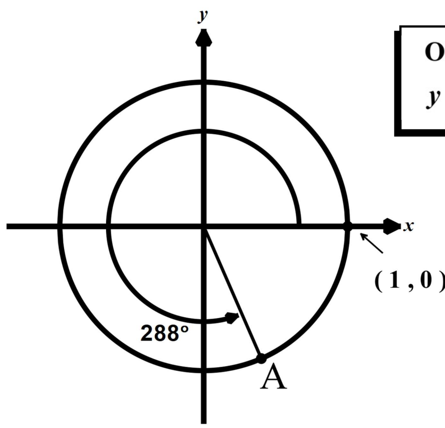
3. Point *A* lies on the unit circle at an angle of 288° as shown in the diagram. Which of the following is the *y*-coordinate of point *A*?

(1) -0.31

(2) -0.63

(3) -0.86

(4) -0.95



On the unit circle $y-\text{coord} = \sin\theta$

$$y = \sin(288°) = -0.951... \approx -0.95$$

(4)

4. An angle θ drawn in standard position terminates in the second quadrant. If $\sin\theta = \frac{4}{7}$, then which of the following is the value of $\cos\theta$?

(1) $-\frac{\sqrt{33}}{7}$ (3) $-\frac{3}{7}$

(2) $-\frac{\sqrt{33}}{4}$ (4) $\frac{\sqrt{3}}{2}$

$$\cos^2\theta + \left(\frac{4}{7}\right)^2 = 1$$

$$\cos^2\theta + \frac{16}{49} = 1$$

$$\cos^2\theta = \frac{33}{49} \Rightarrow \cos\theta = \pm\sqrt{\frac{33}{49}} = \pm\frac{\sqrt{33}}{7}$$

$$\cos\theta = -\frac{\sqrt{33}}{7}$$ **since cosine is negative in II**

(1)

5. On a cosine function $y = A\cos(Bx) + C$, the maximum value is 32 and the minimum value is -4. Which of the following must be the value of $|A| - C$?

(1) -4 (3) -3

(2) 8 (4) 4

$$|A| = \frac{32-(-4)}{2} = \frac{36}{2} = 18$$
$$C = \frac{32+-4}{2} = \frac{28}{2} = 14$$
$$|A| - C = 18 - 14 = 4$$

(4)

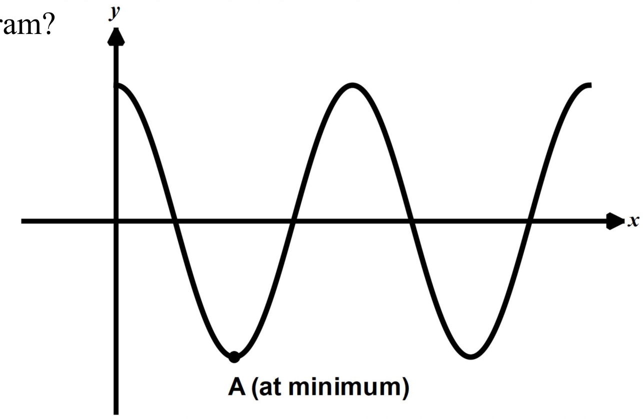
6. The graph of $y = k\cos\left(\frac{\pi}{4}x\right)$ is shown below for unknown constant k. What is the x-coordinate of point A shown marked on the diagram?

(1) $\frac{3k}{4}$

(2) π

(3) $\frac{\pi k}{2}$

(4) 4



$$\frac{\pi}{4}P = 2\pi$$
$$P = 2\pi \cdot \frac{4}{\pi} = 8$$
$$x-\text{coord} = \frac{1}{2}P = 4$$

(4)

7. The distance, h, in feet above the ground of a point on the circular frame of a Ferris Wheel is given by $h = 42\cos(12t) + 46$, where t is the number of seconds since the wheel started turning. The diameter of the wheel is

(1) 42 (3) 84

(2) 46 (4) 88

$$\text{Diameter} = 2A = 2(42) = 84$$
or
$$h_{max} = 46 + 42 = 88$$
$$h_{min} = 46 - 42 = 4$$
$$\text{Diameter} = 88 - 4 = 84$$

(3)

8. At which of the following angles is the secant function undefined?

(1) 90° (3) 240°

(2) 180° (4) 330°

$$\sec A = \frac{1}{\cos A}$$
$$\cos 90° = 0$$
$\sec 90°$ is undefined

(1)

9. If $0 \le \theta \le 90^\circ$ and $\sin(\theta) = p$, then which of the following gives the value of $\tan(\theta)$?

(1) $\frac{p}{1-p}$ (3) $\frac{p}{\sqrt{1-p^2}}$

(2) $\frac{\sqrt{1-p^2}}{p}$ (4) $\sqrt{1-p^2}$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$
$$\cos^2(\theta) + \sin^2(\theta) = 1$$
$$\cos(\theta) = \sqrt{1-\sin^2(\theta)}$$

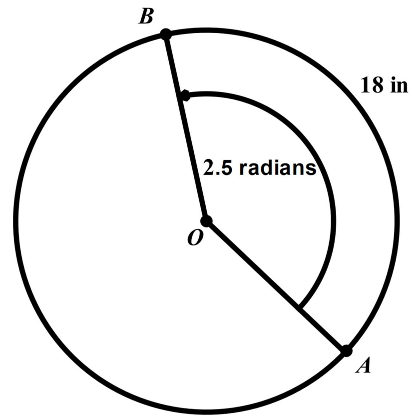
⇒

$$\sin(\theta) = p$$
$$\cos(\theta) = \sqrt{1-p^2}$$
$$\tan(\theta) = \frac{p}{\sqrt{1-p^2}}$$

(3)

PART II QUESTIONS: Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps and explain your reasoning. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

10. In the circle shown graphed below, an 18 inch arc is traced from point *A* to point *B* such that a radian angle of 2.5 is rotated through. What is the radius of the circle in inches?



$$\theta = \frac{s}{r} \Rightarrow 2.5 = \frac{18}{r} \Rightarrow 2.5r = 18$$
$$r = \frac{18}{2.5} = 7.2$$

11. Point *B* has coordinates $(7, -24)$ and lies on the circle whose equation is $x^2 + y^2 = 625$. If an angle is drawn in standard position with its terminal ray extending through point *B*, what is the sine of the angle?

$$\sin A = \frac{y-\text{coord}}{\text{radius}} = \frac{-24}{\sqrt{625}} = -\frac{24}{25} \text{ or } -0.96$$

12. For angle *A* it is known that $\sin(A) > 0$ and $\tan(A) < 0$. If *A* is drawn in standard position, in which quadrant does its terminal ray lie?

$\sin(A) > 0 \Rightarrow \angle A$ must be in Quads I or II

$\tan(A) < 0 \Rightarrow \angle A$ must be in Quads II or IV

⇨ **$\angle A$ must terminate in Quad II**

13. The function $y = -6\sin(x) + C$ has a maximum value of 10. What is its minimum value? Explain your answer.

We can find the value of *C* by using the maximum and the amplitude, $|A|$.

⇨ $y_{max} = C + |A|$, $10 = C + 6$, $C = 4$

⇨ $y_{min} = C - |A|$, $y_{min} = 4 - 6$, $y_{min} = -2$

14. Roger is on a playground swing, and he is swinging back and forth in such a way that the height, *h*, in feet, of the swing off the ground is given by the equation $h = 3\cos\left(\frac{3\pi}{2}t\right) + 5$, where *t* is in seconds. How many seconds elapses between two consecutive times that the swing is at its maximum height?

In this question we need to find the <u>period</u> of the function.

⇨ $BP = 2\pi$, $\frac{3\pi}{2}P = 2\pi$

⇨ $P = 2\pi \cdot \frac{2}{3\pi}$, $P = \frac{4}{3}$ seconds

PART III QUESTIONS: Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps and explain your reasoning. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

15. Given the graph of $y = A\cos(Bx) + C$ shown below and the two points marked, determine the values of A, B, and C. Show how you arrived at your answers.

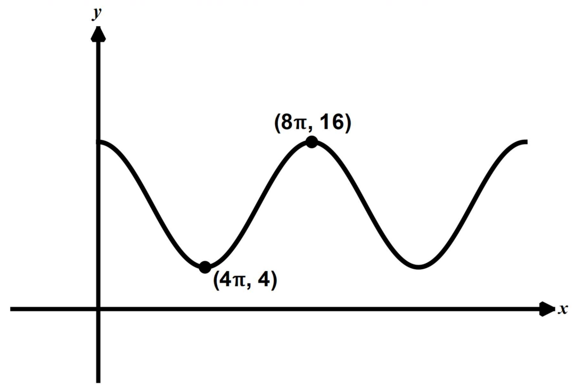
$A = \frac{16-4}{2} = \frac{12}{2} = 6$

$C = \frac{4+16}{2} = \frac{20}{2} = 10$

$P = 8\pi$

$8\pi B = 2\pi$

$B = \frac{2\pi}{8\pi} = \frac{1}{4}$



16. If $90^\circ \le \theta \le 180^\circ$ and $\sin(\theta) = \frac{5}{12}$ then determine the exact values of $\cos(\theta)$ and $\tan(\theta)$.

$\cos^2(\theta) + \sin^2(\theta) = 1$

$\cos^2(\theta) + \left(\frac{5}{12}\right)^2 = 1$

$\cos^2(\theta) + \frac{25}{144} = 1$

$\cos^2(\theta) = 1 - \frac{25}{144}$

$\cos^2(\theta) = \frac{119}{144}$

⇨

$\cos(\theta) = \pm\sqrt{\frac{119}{144}} = \pm\frac{\sqrt{119}}{12}$

$\cos(\theta) = -\frac{\sqrt{119}}{12}$

$\cos(\theta) < 0$ since θ is in Quad II

⇨

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{5/12}{-\sqrt{119}/12}$

$\tan(\theta) = -\frac{5}{\sqrt{119}}$ or

$\tan(\theta) = -\frac{5\sqrt{119}}{119}$

17. The temperature inside a parked car in a long-term parking lot, in degrees Fahrenheit, can be modeled using the equation $F(t) = -28\cos\left(\frac{\pi}{12}t\right) + 84$, where t is the number of hours since 3:00 a.m.

(a) Determine the maximum inside temperature of the car. Explain how you arrived at your answer.

The maximum will simply be the amplitude, 28, added to the midline value, 84.

⇨

$F_{max} = 84 + 28 = 112$ °F

(b) At what time of day does the inside temperature of the car attain its maximum? Explain how you arrived at your answer.

The maximum will be a half-period past the minimum, where the curve starts at 3:00 a.m. So we first find the period, *P*.

⇨

$\frac{\pi}{12}P = 2\pi$

$P = 2\pi \cdot \frac{12}{\pi} = 24$ hours

⇨

So, the maximum is reached 12 hours after 3:00 am so it must occur at 3 p.m.