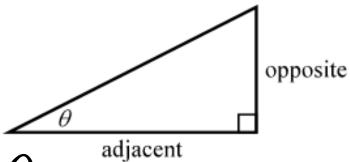
Name: ______________________________ Date: ________________

The Tangent Function
Common Core Algebra II

The two most important circular functions are **sine** and **cosine**. But, recall from Common Core Geometry, that there was a third trigonometric function known as **tangent**. There are actually three more that you will learn about in the next lesson, but in this lesson we will only look at the tangent function. Let's recall how it was defined using **right triangle trigonometry**.

Right Triangle Trigonometry Definition of Tangent

$$\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}}$$



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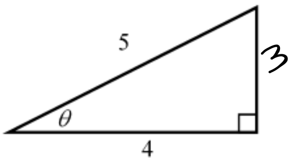
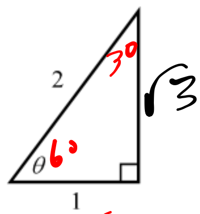
***Exercise* #1:** For each of the right triangles below, state the values of $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.

(a)



$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

(b)



$1^2 + b^2 = 2^2$

$1 + b^2 = 4$

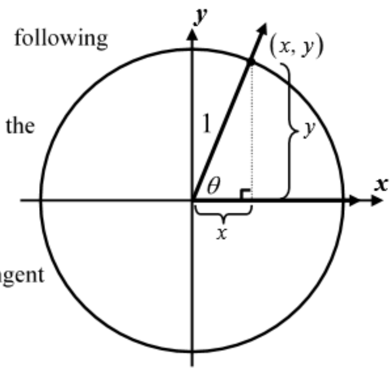
$b^2 = 3$

$b = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\cos\theta = \frac{1}{2}$ $\tan\theta = \frac{\sqrt{3}}{1}$

Notice that we can have missing sides of a right triangle and still use the Pythagorean Theorem to find these ratios (many of which may involve radicals and thus irrational numbers). Just as we did with sine and cosine, we can also define tangent in terms of the **unit circle**.

***Exercise* #2:** Consider the unit circle shown below. Answer the following questions based on this diagram.



(a) If (x, y) represents a point on the circle, how can we define the tangent function in terms of this point?

(b) Given how we define sine and cosine, how can we define the tangent function in terms of sine and cosine?

Common Core Algebra II, Unit #11 – The Circular Functions – Lesson #10
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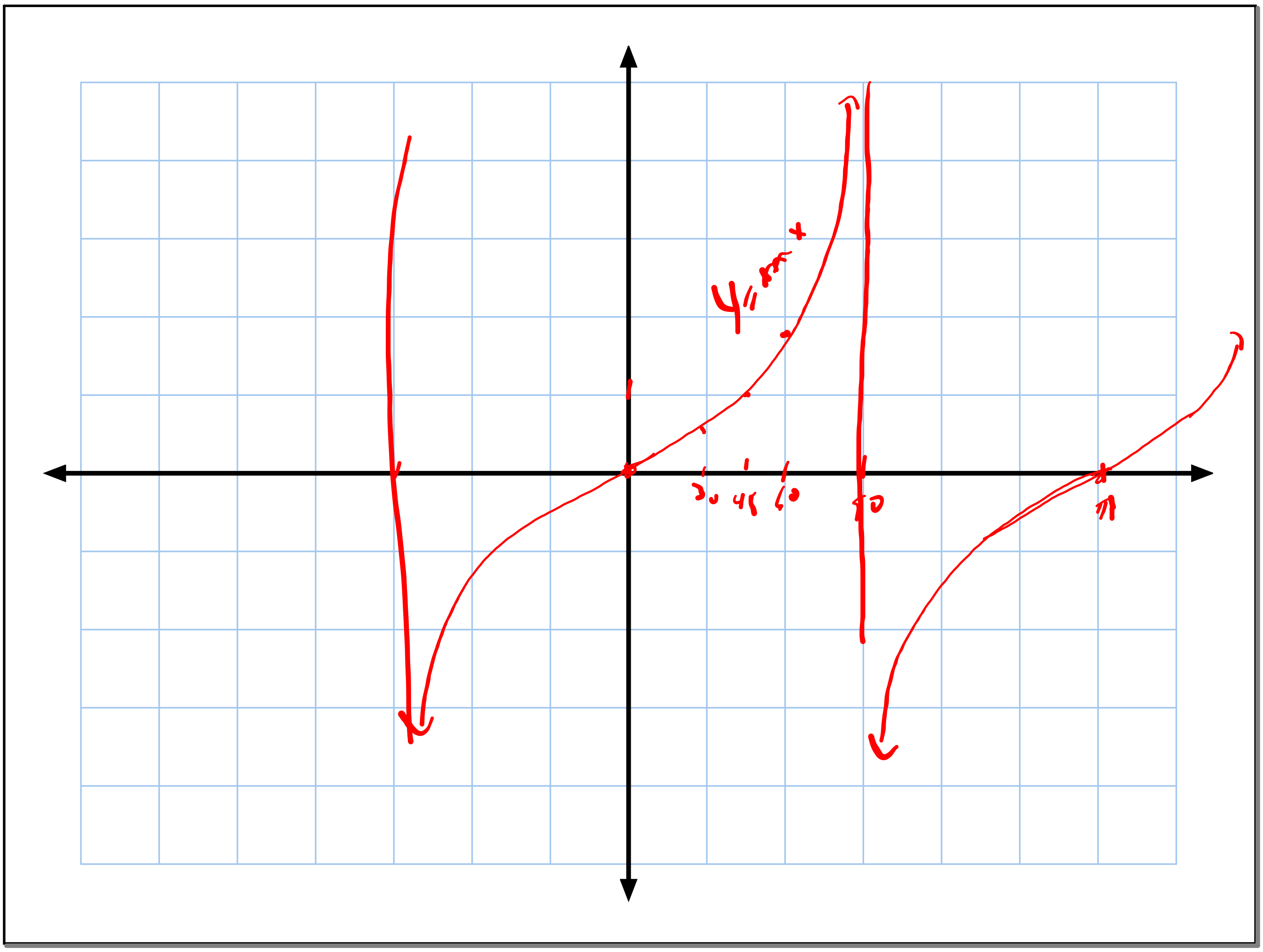
30
45
60
90

THE DEFINITION OF TANGENT IN TERMS OF SINE AND COSINE

$$\tan\theta = \frac{\sin(\theta)}{\cos(\theta)} = \frac{y}{x}$$

***Exercise* #3:** Use the unit circle and the definition of tangent in terms of sine and cosine to find the value for each of the following. Do not leave any complex fractions.

(a) $\tan(60^\circ) = \frac{\sin 60}{\cos 60}$ $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

(b) $\tan(45^\circ) = \frac{\sin 45}{\cos 45}$ $= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

(c) $\tan(150^\circ) = \frac{\sin 30}{-\cos 30}$ $\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{2}\left(\frac{-2}{\sqrt{3}}\right) = \frac{-1}{\sqrt{3}}\frac{(\sqrt{3})}{(\sqrt{3})}$ $\frac{-\sqrt{3}}{3}$

(d) $\tan(180^\circ)$ $\frac{0}{-1} = 0$

Because the tangent function involves division, there is a chance it could be undefined.

***Exercise* #4:** Consider the angle $\theta = 90^\circ$ or $\frac{\pi}{2}$ radians.

(a) State the values of sine and cosine at this angle.

$\tan 90 = \frac{\sin 90}{\cos 90} = \frac{1}{0} = u$

(b) Why would the value of $\tan(90^\circ)$ be undefined?

b/c we cannot divide by 0.

***Exercise* #5:** At which of the following angles is tangent undefined?

(1) $\theta = 0^\circ$
(2) $\theta = 270^\circ$
(3) $\theta = 120^\circ$
(4) $\theta = -180^\circ$

On a final note, it is interesting that if we know sine or cosine of an angle and the quadrant of the angle we can find the other two missing trigonometric values.

***Exercise* #6:** Determine the value of $\cos(\theta)$ and $\tan(\theta)$ if $\sin(\theta) = \frac{5}{13}$ and the terminal ray of θ lies in the second quadrant.

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$\cos\theta = \frac{-12}{13}$ $\tan\theta = \frac{5}{-12}$

TOA CAH $\cos\theta = \frac{a}{1}$

4th

***Exercise* #7:** If $\cos(\theta) = a$, where $a > 0$, and the terminal ray of θ lies in the fourth quadrant, then which of the following gives the value of $\tan(\theta)$ in terms of a.

$\frac{-y}{x}$

(1) $\frac{1-a}{a}$
(2) $-\sqrt{1-a^2}$
(3) ~~$\sqrt{1-a^2}$~~
(4) $\frac{-\sqrt{1-a^2}}{a}$

tan 4th is −

$-\sqrt{1-a^2}$

$a^2 + x^2 = 1$
$x^2 = 1 - a^2$
$x = \sqrt{1-a^2}$

Name: ______________________________ Date: ________________

THE TANGENT FUNCTION
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Using the unit circle diagram, find the exact values for each of the following. Don't leave any complex fractions. Show how you arrived at your final answers. You can check using your calculator, but decimal answers should not be given. If the value of tangent is undefined, state UND.

(a) $\tan(60^\circ)$

$\sqrt{3}$

(b) $\tan(150^\circ) = -\tan 30$

$-\frac{\sqrt{3}}{3}$

(c) $\tan(225^\circ) = +\tan 45$

1

(d) $\tan(270^\circ)$

U

(e) $\tan\left(\frac{2\pi}{3}\right) = -\tan 60$

$\frac{2\pi}{3}\left(\frac{180}{\pi}\right) = 120$

$-\sqrt{3}$

R→D

(f) $\tan\left(\frac{11\pi}{6}\right) = -\tan 30$

$\frac{11\pi}{6}\left(\frac{180}{\pi}\right) = 330$

$-\frac{\sqrt{3}}{3}$

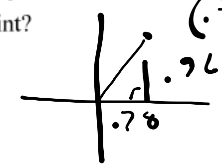
2. The point $(0.28, 0.96)$ lies on the unit circle. Which of the following is closest to the tangent of an angle drawn in standard position whose terminal ray passes through this point?

(1) 3.43 (3) 0.29

(2) 1.73 (4) 0.42

$\tan\theta = \frac{.96}{.28}$

(.28, .96)



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3. At which of the following angles is the tangent function undefined? 90, 270 . . . .

(1) ~~$\theta = 180^\circ$~~ (3) ~~$\theta = 45^\circ$~~

(2) $\theta = -90^\circ$ (4) ~~$\theta = -360^\circ$~~

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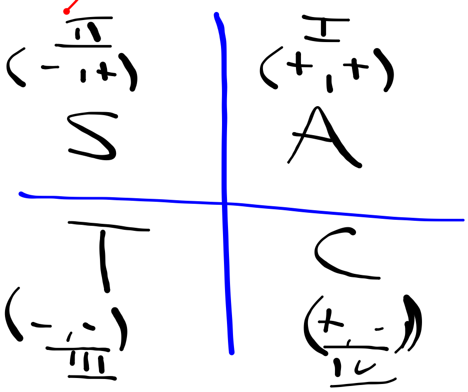
4. Which of the following values of x is *not* in the domain of $g(x) = \tan(2x)$? Hint – you will be multiplying each of these values by 2 before finding its tangent.

(1) 45° (3) 180°

(2) 0° (4) 90°

____

θ	0	30	45	60	90
$\sin\theta$ y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$ x	1	$\sqrt{3}/2$	$\frac{\sqrt{2}}{2}$	$1/2$	0
$\tan\theta$ $\frac{y}{x}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	u



All students take calc.

5. Determine whether each function in the tables below is positive, $(+)$, or negative, $(-)$, for angles whose terminal rays lie in the respective quadrants. Use values of sine and cosine to determine the sign of the tangent function.

S | A
T | C

	I	II	III	IV
$\cos(\theta)$	+	−	−	+
$\sin(\theta)$	+	+	−	−
$\tan(\theta)$	+	−	+	−

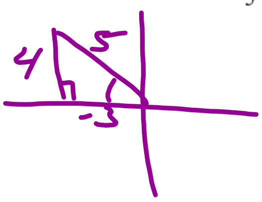
6. For an angle α it is known that $\tan(\alpha) > 0$ (+) and $\sin(\alpha) < 0$ (−). The terminal ray of α when drawn in standard position must lie in which quadrant? Hint: see the table in #5 for help.

(1) I (3) III

(2) II (4) IV

7. For each of the following problems, the value of either sine or cosine of an angle is given along with the quadrant in which the terminal ray of the angle lies. For each, produce the values of the two missing trigonometric functions. Some of your answers will have radicals (irrational numbers) in them. You should *not* leave complex fractions.

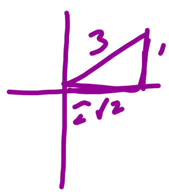
(a) $\cos(A) = \frac{-3}{5}$ and A terminates in quadrant II.



$\sin A = \frac{4}{5}$

$\tan A = \frac{4}{-3}$

(b) $\sin(\alpha) = \frac{1}{3}$ and α terminates in quadrant I.



$\cos A = \frac{2\sqrt{2}}{3}$

$1^2 + b^2 = 3^2$

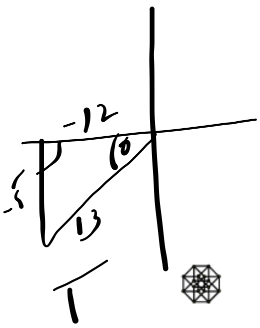
$b^2 = 8$

$b = \sqrt{8}$

$b = 2\sqrt{2}$

$\tan A = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

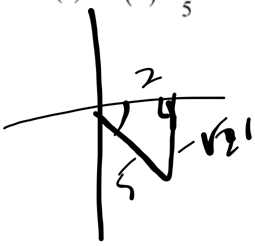
(c) $\sin(\theta) = -\frac{5}{13}$ and θ terminates in quadrant III.



$\cos\theta = \frac{-12}{13}$

$\tan\theta = \frac{5}{12}$

(d) $\cos(B) = \frac{2}{5}$ and B terminates in quadrant IV.



$2^2 + b^2 = c^2$

$b^2 = 21$

$b = \sqrt{21}$

$\sin\theta = \frac{-\sqrt{21}}{5}$

$\tan\theta = \frac{-\sqrt{21}}{2}$