Name: ______________________________ Date: ______________

@ origin
$x^2+y^2=r^2$

THE UNIT CIRCLE
COMMON CORE ALGEBRA II

@ pt (h,k) center
$(x-h)^2+(y-k)^2=r^2$
changing of signs

The basis of trigonometry will be a very special circle known as **the unit circle**. This is simply a circle that has its center located at the origin and has a radius equal to one unit (hence the name "unit").

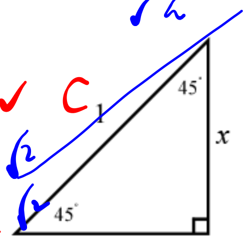
***Exercise* #1:** From our work with equations of circles, which of the following would represent the equation of $\boxed{\text{the unit circle?}}$ — Centered @ origin w/ radius of 1.

(1) $x+y=1$ (3) $x^2+y^2=1$

(2) $y=x^2+1$ (4) $(x-1)^2+(y-1)^2=1$ ______

Next we will seek to produce some of the coordinate points that lie on the unit circle through the use of the Pythagorean Theorem. The next two exercises will illustrate the important right triangles we will need.

***Exercise* #2:** Consider the right triangle shown whose hypotenuse is equal to one and whose angles are both equal to 45°. Since this is an isosceles right triangle, the two equal sides are labeled x. Solve for x and place your answer in simplest radical form.

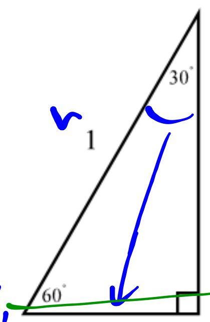


Pyth. Theorem
$a^2+b^2=c^2$

$x^2+x^2=1^2$
$2x^2=1$
$x^2=\frac{1}{2}$
$x=\sqrt{\frac{1}{2}}$ ✓
$\sqrt{\frac{1}{2}}=\frac{\sqrt{1}}{\sqrt{2}}=\frac{1}{\sqrt{2}}$
$x=\frac{\sqrt{2}}{2}$

***Exercise* #3:** Consider the 30°-60° right triangle shown whose hypotenuse is equal to one. Clearly this triangle is half of an equilateral triangle.

SOH CAH TOA



(a) What is the length of the shorter side of this right triangle?

$\sin 30=\frac{x}{1}$ $x=\sin 30$
$x=\frac{1}{2}$

(b) Using the Pythagorean Theorem, find the length of the longer side in simplest radical form.

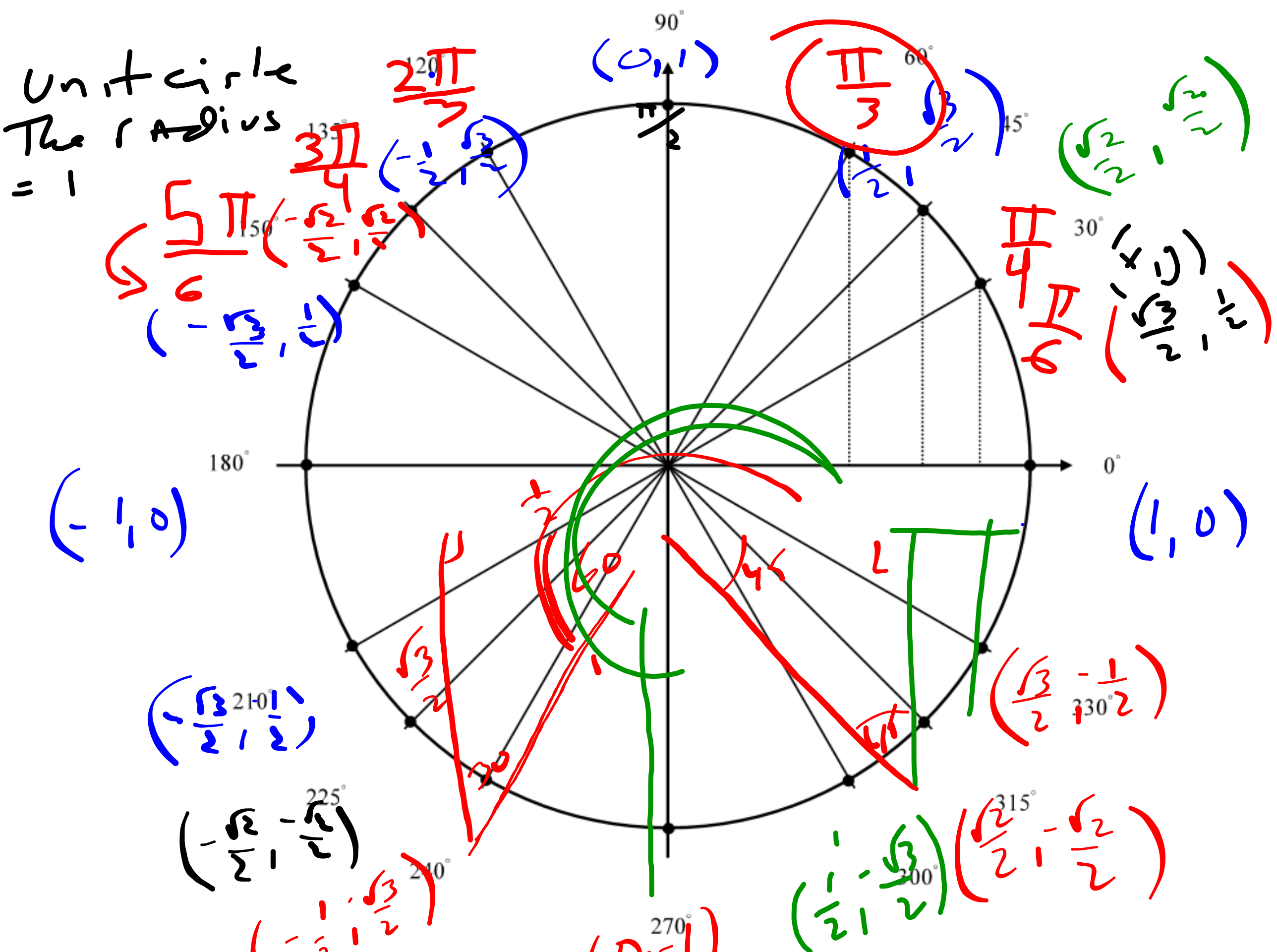
$a^2+b^2=c^2$
$\left(\frac{1}{2}\right)^2+b^2=1^2$
$\frac{1}{4}+b^2=1$
$-\frac{1}{4}$ $-\frac{1}{4}$
$b^2=\frac{3}{4}$
$b=\sqrt{\frac{3}{4}}$
$b=\frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{\sqrt{4}}=\frac{\sqrt{3}}{2}$

$120\left(\frac{\pi}{180}\right)=\frac{2\pi}{3}$ $135\left(\frac{\pi}{180}\right)=\frac{3\pi}{4}$ $150\left(\frac{\pi}{180}\right)\frac{5\pi}{6}$

***Exercise* #4:** The diagram below represents the **unit circle**. Based on your work from *Exercises* #2 and #3, fill in the ordered pairs at each of the following angles that are assumed to be drawn in standard position.

Unit circle
The radius = 1

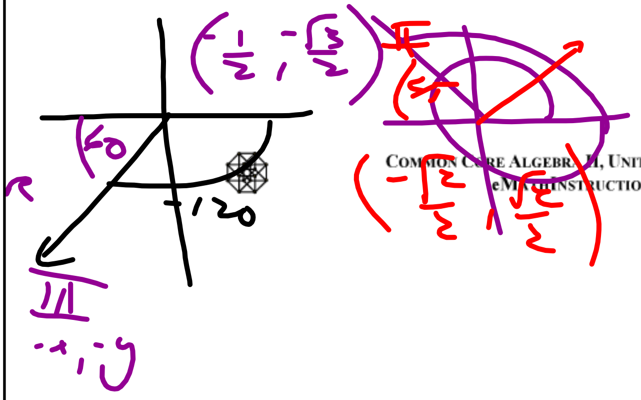


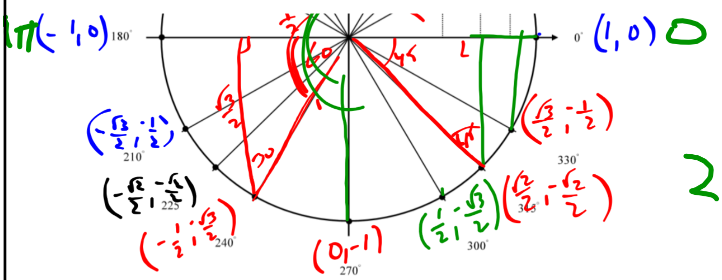
***Exercise* #4:** For each of the following angles drawn in standard position, give the coordinate pair from the unit circle.

(a) -120° (b) 495° (c) $\frac{\pi}{3}$ (d) $\frac{3\pi}{2}$

135° 270°

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $(0, -1)$





$270\left(\frac{\pi}{180}\right) \frac{3\pi}{2}$

$210\left(\frac{\pi}{180}\right) = \frac{7\pi}{6}$ $225\left(\frac{\pi}{180}\right) = \frac{5\pi}{3}$ $240\left(\frac{\pi}{180}\right) \frac{4\pi}{3}$

$330\left(\frac{\pi}{180}\right) = \frac{11\pi}{6}$

$315\left(\frac{\pi}{180}\right) = \frac{7\pi}{4}$ $300\left(\frac{\pi}{180}\right) = \frac{5\pi}{3}$

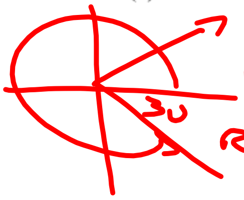
Name: ________________________________ Date: ________________

The Unit Circle
Common Core Algebra II Homework

Fluency

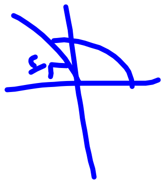
1. Draw a rotation diagram for each of the following angles and then determine the ordered pair that lies on the unit circle for each angle.

(a) $\theta = 330^\circ$

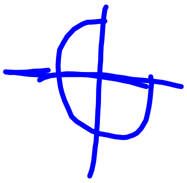


$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

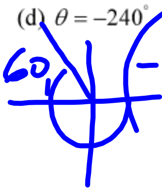
(b) $\theta = 135^\circ$ $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$



(c) $\theta = -270^\circ$ $(0, 1)$

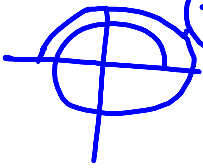


(d) $\theta = -240^\circ$



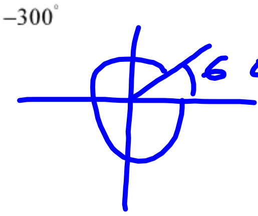
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(e) $\theta = 540^\circ$



$(-1, 0)$

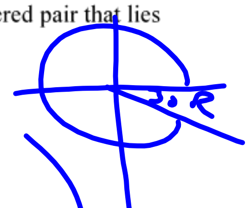
(f) $\theta = -300^\circ$



$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

2. Draw a rotation diagram for each of the following radian angles and then determine the ordered pair that lies on the unit circle for each angle.

$-270^\circ \Rightarrow 90$ 330°



(a) $\alpha = \frac{2\pi}{3}$ 120°

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

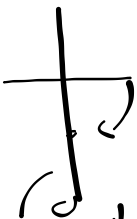
(b) $\alpha = -\frac{3\pi}{2}$

$(0, 1)$

(c) $\alpha = \frac{11\pi}{6}$

$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

(d) $\alpha = -\frac{\pi}{2}$



$(0, -1)$

(e) $\alpha = \frac{3\pi}{4}$

$\frac{3\pi}{4}\left(\frac{180}{\pi}\right) = 135$

R 45 $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

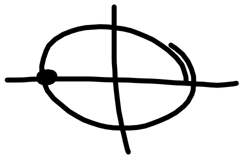
(f) $\alpha = \frac{4\pi}{3}$ 240° R 30°

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

3. All of the points on the unit circle must satisfy the equation $x^2 + y^2 = 1$. Verify that this equation is true for each of the coordinate points given below.

yes

(a) $(-1, 0)$



(b) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

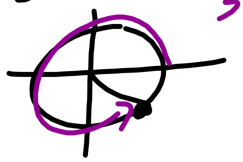
yes

$60°$

(c) $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

yes

$315°$



4. There are other points on the unit circle besides the ones that we determined in this lesson. Every point, though, must satisfy the equation $x^2 + y^2 = 1$. For each of the following problems, either the x or y coordinate of a point on the unit circle is given. Find all possibilities for the other coordinate for this point using the unit circle equation.

2 points

(a) $x = \frac{3}{5}$

$\left(\frac{3}{5}\right)^2 + y^2 = 1$

$\frac{9}{25} + y^2 = 1$

$y = \pm\frac{4}{5}$

$\left(\frac{3}{5}, \frac{4}{5}\right)$ $\left(\frac{3}{5}, -\frac{4}{5}\right)$

(b) $y = -\frac{5}{13}$

$x^2 + \left(-\frac{5}{13}\right)^2 = 1$

$x^2 + \frac{25}{169} = 1$

$\sqrt{x^2} = \sqrt{\frac{144}{169}}$

$x = \pm\frac{12}{13}$

$\left(\frac{12}{13}, -\frac{5}{13}\right)$ $\left(-\frac{12}{13}, -\frac{5}{13}\right)$

(c) $x = \frac{1}{4}$

$\left(\frac{1}{4}\right)^2 + y^2 = 1$

$\frac{1}{16} + y^2 = 1$

$y^2 = \frac{15}{16}$

$y = \pm\frac{\sqrt{15}}{4}$

$\left(\frac{1}{4}, \frac{\sqrt{15}}{4}\right)$ $\left(\frac{1}{4}, -\frac{\sqrt{15}}{4}\right)$

5. For each of the following angles, determine its reference angle. Then state the coordinate on the unit circle for **both** the angle and its reference. What do you notice about the coordinate pairs?

(a) $\theta = 150°$

$R = 30$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(b) $\theta = 225°$

$R\ 45°$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

(c) $\theta = 300°$

$R\ 60$

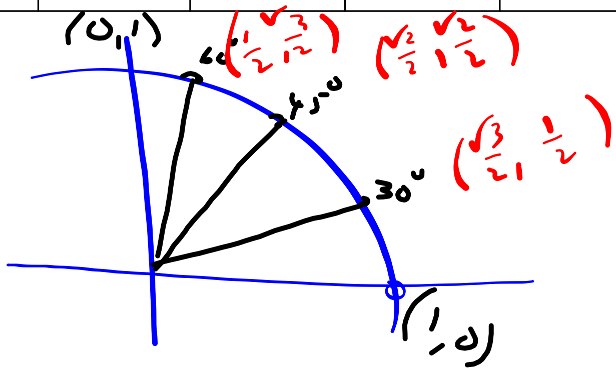
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

	θ	0	30	45	60	90
y	sin θ	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
x	cos θ	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
$\frac{y}{x}$	tan θ					

II $(-,+)$ I $(+,+)$

III $(-,-)$ IV $(+,-)$



	θ	0	30	45	60	90
y	sin θ	0	$1/2$	$\frac{\sqrt{2}}{2}$	$\sqrt{3}/2$	1
x	cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$1/2$	0
$\frac{y}{x}$	tan θ					